

Dual description of QCD*

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It is demonstrated that the field strength approach to Yang Mills theories has essential features of the dual description. In D=3 this approach is formulated in terms of gauge invariant variables.

1 Introduction

According to 't Hooft and Mandelstam¹ confinement may be realized as dual Meissner effect. This confinement scenario assumes that the QCD ground state consists of a condensate of magnetic monopoles (dual superconductor), which squeezes the color electric field of color charges into flux tubes. This scenario has indeed received support from lattice calculations².

Obviously the dual Meissner effect can be most efficiently described in a dual formulation, which is known to exist for quantum electrodynamics. The transition to the dual theory basically amounts to an interchange of the electric and the magnetic fields. At the same time the coupling constant is inverted. In non-Abelian gauge theories duality was considered first by Montone and Olive³ who conjectured that solitons of the original gauge theory become massive fields in the dual theory. This idea was taken up by Seiberg and Witten⁴ who studied duality in supersymmetric theories. By showing that certain supersymmetric models are dual to each other they succeeded to find exact solutions to the strong coupling regime of some supersymmetric gauge theories.

Obviously the strong coupling regime of QCD could be most efficiently studied within the dual theory. Unfortunately the dual theory of non-supersymmetric Yang-Mills theory and in particular of QCD is not known and perhaps does not exist in the strict sense. For this reason there have been attempts to construct the dual theory of QCD phenomenologically⁵. There is also a microscopic approach to this problem which has not been fully appreciated in the past. This is the so-called field strength approach⁶ which formulates the Yang-Mills theory in terms of field strengths. In my talk I would like to demonstrate that this approach in fact yields a formulation of Yang-Mills theory which comes very close to a dual description.

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2 The field strength approach as dual formulation of Yang-Mills theory

Consider the standard functional integral formulation of Yang-Mills theory

$$Z[j] = \int DA_\mu \delta_{gf} \exp \left[-\frac{1}{4\kappa^2} \int (F(A))^2 + \int jA \right] . \quad (1)$$

Here $A_\mu(x)$ denotes the gauge field, j is an external source and δ_{gf} is a short notation for the gauge fixing. Furthermore $F_{\mu\nu}^a(A) = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + f^{abc} A_\mu^b A_\nu^c$ is the field strength where f^{abc} denotes the structure constant of the gauge group $SU(N)$ and κ is the coupling constant. Let us emphasize that due to the gauge fixing the measure of the functional integral is not flat. In fact recently it was explicitly demonstrated that the standard Faddeev-Propov gauge fixing yields precisely the required Haar-measure for the gauge invariant partition function⁷ (see also ref. 8).

In the field strength approach the Yang-Mills action is linearized by means of an auxiliary tensor field $\chi_{\mu\nu}^a(x)$, which has the structure of a field strength, by the identity⁶

$$\exp \left[-\frac{1}{4\kappa^2} \int (F(A))^2 \right] = \int D\chi_{\mu\nu}^a \exp \left[-\frac{\kappa^2}{4} \int \chi^2 + \frac{i}{2} \int \chi F(A) \right] . \quad (2)$$

Inserting this identity into the functional integral (1) and casting the gauge fixing constraint from the gauge potential to the tensor field $\chi_{\mu\nu}^a(x)$ one can integrate out the gauge field explicitly leaving an effective tensor theory defined by

$$Z[j] = \int D\chi_{\mu\nu} (\det \hat{\chi})^{-1/2} \exp [-S_{FS}(\chi) - S_j(\chi)] . \quad (3)$$

Here the functional determinant of the matrix $\hat{\chi}_{\mu\nu}^{ab} = f^{abc} \chi_{\mu\nu}^c$ arises from the Gaussian integral over the gauge field. The action of the tensor field

$$S_{FS}(\chi) = \frac{\kappa^2}{4} \int \chi^2 + \frac{i}{2} \int \chi F(V) \quad (4)$$

is just the exponent of the right-hand side of equation (2) taken, however, at the stationary phase value of the gauge potential, which is given by the induced gauge potential

$$V_\mu^a = (\hat{\chi}^{-1})_{\mu\nu}^{ab} \partial_\lambda \chi_{\nu\lambda}^b . \quad (5)$$

This induced gauge potential behaves under gauge transformation like the original gauge field $A_\mu(x)$. Finally,

$$S_j(\chi) = \int jV + \frac{i}{2} \int j\hat{\chi}^{-1}j \quad (6)$$

contains the dependence on the external source j .

Let us emphasize that (3) is an exact representation of the initial Yang-Mills functional integral equation (1). A few comments are here in order. From electrodynamics we know that certain phenomena in topologically non-connected spaces cannot be described exclusively in terms of the field strength but are sensitive to the (at least topological) properties of the gauge potential, as in the case of the Bohm-Aharonov effect. Therefore one might wonder how the field strength formulation (3) can be an equivalent representation of the Yang-Mills theory (1). In fact, due to the presence of the induced gauge potential (5), which in fact couples to the external source j in the same way as the initial gauge potential, the field strength approach (3) is also capable of describing those phenomena.

Let us compare now the field strength formulation with the standard formulation. In the standard formulation we start with the gauge potential $A_\mu(x)$ and construct from this potential the field strength $F_{\mu\nu}(A)$. By construction this field strength then satisfies the Bianchi identity

$$[D_\mu(A), \tilde{F}_{\mu\nu}(A)] = 0 , \quad (7)$$

where

$$D_\mu(A) = \partial_\mu + A_\mu , \quad \tilde{F}_\mu = \frac{1}{2}\epsilon_{\mu\nu k\gamma}F_{k\lambda} . \quad (8)$$

By minimizing the Yang-Mills action

$$S_{YM} = \frac{1}{4k^2} \int (F(A))^2 \quad (9)$$

one finds the classical Yang-Mills equation of motion

$$[D_\mu(A), F_{\mu\nu}] = 0 , \quad (10)$$

the solutions of which are the well-known instantons.

On the other hand, in the field strength approach the fundamental quantity is the tensor field $\chi_{\mu\nu}^a(x)$ and from this the induced gauge field (5) is built up. By construction this induced gauge field satisfies the equation of motion

$$[D_\mu(V), F_{\mu\nu}] = 0 \quad (11)$$

but will in general not satisfy the Bianchi identity. However, the classical equation of motion obtained by minimizing the action (4) reads

$$\chi = iF_{\mu\nu}(V) , \quad (12)$$

which shows that the classical tensor fields are in turn (up to a constant) equivalent to the field strength constructed from the induced gauge field (5). This implies that the classical tensor fields in fact satisfy the Bianchi identity

$$[D_\mu(V), \tilde{\chi}_{\mu\nu}] = 0 . \quad (13)$$

Therefore we observe that in the field strength approach the roles of the classical equation of motion and the Bianchi identity are interchanged compared to the original Yang-Mills theory. This is an essential feature of a dual formulation. Moreover, as it is clear from equation (2), in the field strength formulation the coupling constant is inverted, compared to the original theory. This, together with the above observation justifies calling the field strength approach the dual formulation of Yang-Mills theory, although it is not formulated in terms of a dual potential. This, however, might be an advantage.

Let us also mention, if one applies the field strength approach to QED one obtains in fact the dual QED. Furthermore, for compact QED the field strength approach yields the \mathbf{Z} gauge theory.

The semiclassical analysis of Yang-Mills theory can be performed in the field strength approach in the same way as in the standard formulation. In fact the stationary points of the field strength action (4) are given by $iF_{\mu\nu}[A_\mu^{\text{inst}}]$ where A^{inst} denotes the instanton gauge potential and the corresponding induced gauge field (5) becomes the instanton field. Furthermore, calculation of the leading quantum fluctuations⁹ yields the same result as in the standard approach.

3 Field strength approach to D=3 Yang-Mills theory in gauge invariant variables

In D=3 dimensions the tensor field can be expressed in terms of a color vector

$$\chi_{ij} = \epsilon_{ijh} \chi_k^a \quad (14)$$

which transforms homogeneously under gauge transformations. In terms of the color vector χ_k^a the induced gauge potential (5) becomes precisely the representation of the gauge potential introduced by Johnson et al. in their gauge invariant formulation of Yang-Mills theory in the hamilton approach¹⁰.

We can use their result to formulate the field strength approach to three-dimensional Yang-Mills theory in terms of gauge invariant variables. For this purpose we introduce the gauge invariant metric

$$g_{ij} = \chi_i^a \chi_j^a . \quad (15)$$

In the same way as in gravity we introduce the affine connection

$$\Gamma_{jk} = \frac{1}{2} g^{im} (\partial_j g_{mk} + \partial_k g_{jm} - \partial_m g_{jk}) . \quad (16)$$

From this we construct the Riemann curvature

$$R_{kij}^\ell = \partial_i \Gamma_{jh}^\ell - \partial_j \Gamma_{ik}^\ell + \Gamma_{jk}^m \Gamma_{im}^\ell - \Gamma_{ik}^m \Gamma_{jm}^\ell . \quad (17)$$

Further, defining the Ricci curvature and the corresponding Ricci scalar

$$R_{k\ell} = R_{kil} , \quad R = R_k^k , \quad (18)$$

and the Einstein curvature

$$G_{k\ell} = R_{k\ell} - \frac{1}{2} g_{k\ell} R , \quad (19)$$

one can express the field strength of the induced gauge potential by

$$F_{ij}^a(V) = \epsilon_{ijk} B_k^a , \quad B^{ai} = \sqrt{g} \chi_j^a G^{ij} \quad (20)$$

where $\sqrt{g} = \det \chi_i^a$. Using furthermore that in D=3: $\det \hat{\chi} = -2(\det \chi_i^a)^3$, for vanishing external sources $j = 0$ the functional integral (3) can be entirely expressed in terms of the gauge invariant metric (15). One obtains

$$Z[j = 0] = \int \mathcal{D}g_{ij} g^{-2} \exp \left[-\frac{k^2}{2} \int g_{ii} + \frac{i}{2} \int \sqrt{g} g_{ij} G^{ij} \right] . \quad (21)$$

Here the kinetic term of the gauge invariant metric (the last term in the exponent) coincides with the action of D=3 gravity. On the classical level the correspondence between three-dimensional Yang-Mills theory and gravity was also observed in ¹¹.

4 Field strength approach in the Maxwell gauge

The realization of the dual Meissner effect assumes the existence of magnetic monopoles which can be most easily identified by using 't Hooft's Abelian

projection¹². This is based on the Cartan decomposition of the gauge group $G = H \otimes G/H$ where H denotes the Cartan subgroup (invariant torus). Accordingly the gauge potential can be decomposed into a part A_μ^n living in the Cartan subalgebra and a part A_μ^{ch} living in the coset G/H . Lattice calculations¹³ indicate that there is a preferred maximal abelian gauge

$$[\partial_\mu + A_\mu^n, A_\mu^{ch}] = 0. \quad (22)$$

In this gauge the monopoles seem indeed to be the relevant infrared degrees of freedom. One therefore would like to have an effective Abelian theory with magnetic monopoles present, where however the charged field A_μ^{ch} is completely integrated out. This, in fact, can be done in the field strength approach at the expense of an Abelian tensor field. This is achieved by linearizing not the complete field strength as it was done in (2), but only the non-Abelian part of the field strength $[A_\mu^{ch}, A_\nu^{ch}]$. For the gauge group $G = \text{SU}(2)$ this commutator is within the Cartan algebra, and accordingly, the tensor field $\chi_{\mu\nu}$ necessary to linearize the square of this term (see eq. (2) lives also in the Cartan subalgebra. After integrating out the charged gauge field A_μ^{ch} one is left then with an effective theory in the Abelian gauge field A_μ^n and an Abelian tensor field $\chi_{\mu\nu}$. The explicit form of this effective theory has been presented in¹⁴. Before extracting the nonperturbative physics from this effective theory we have calculated the one-loop Beta function¹⁵ and reproduced the standard result. At present we are searching for nontrivial tensor field configurations which would induce interactions between the magnetic monopoles.

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Note added: After this work was completed, Ref. 16 was brought to our attention, which also uses the variables introduced in Ref. 10 to formulate the field strength approach in terms of gauge invariant variables.

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